

Exercise 42

Let $f(x) = \frac{1}{x}$. Find a number c such that the average rate of change of the function f on the interval $(1, c)$ is $-\frac{1}{4}$.

Solution

Set $-1/4$ equal to the average rate of change from $x = 1$ to $x = c$, and solve the resulting equation for c .

$$\begin{aligned} -\frac{1}{4} &= \frac{f(c) - f(1)}{c - 1} \\ &= \frac{\frac{1}{c} - \frac{1}{1}}{c - 1} \\ &= \frac{\frac{1}{c} - \frac{c}{c}}{c - 1} \\ &= \frac{\frac{1-c}{c}}{c - 1} \\ &= \frac{1 - c}{c(c - 1)} \\ &= \frac{-(c - 1)}{c(c - 1)} \\ &= -\frac{1}{c} \end{aligned}$$

Therefore, $c = 4$.